Creep in Soft Soils

W(H)YDOC 05

Paris, 23-25 November, 2005

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Abstract:
The well-known logarithmic creep law for secondary compression is transformed into a differential form in order to include transient loading conditions. This 1-D creep model for oedometer-type strain conditions is then extended towards general 3-D states of stress and strain by incorporating concepts of Modified Cam-Clay and viscoplasticity.

In this way a robust constitutive model is formulated that captures the stress-strain-time behaviour of isotropically consolidated clay samples. Considering lab test data for isotropically consolidated clay samples, it is shown that phenomena such as undrained creep, overconsolidation and aging are well captured by the model.

Both natural and reconstituted clays possess an anisotropic fabric which is coaxial to the direction of sedimentation. This is very pronounced in natural clays, as inter-particle bonds tend to develop during the diagenesis of clay layers. In order to model this anisotropy, one has to depart from the isotropic ellipses of the Modified Cam Clay model, as yield surfaces or contour lines for a constant rate of creep. Using rotated ovals in p-q plane, as justified by data from creep tests, an anisotropic creep model is formulated.
Considering such a rate-type effect, it is important to do research on creep and stress relaxation.
Constant rate of deformation oedometer tests after Marques (1996)

\[ \varepsilon (\%) \]

\[ \sigma' \text{ (kPa)} \]

\[ \varepsilon = \frac{-\Delta e}{1 + e_0} \]


Sällfors (1975): Preconsolidation pressure of soft high plastic clays, Ph. D. Thesis, Chalmers University of Technology, Goteborg, Sweden
Load-controlled test: footing (3.0 x 3.0 m²) on dense sand

Load-controlled tests: cast-in-place concrete micropiles in dense sand

Length: 6.2 to 6.3 m
Diameter: 18 cm

Creep is even observed for piles in dense sand.
Part 1

Creep in oedometer tests

Important publications:


Garlanger (1972): The consolidation of soils exhibiting creep under constant effective stress, Géotechnique 22 (1), pp. 71-78.


1D-Consolidation and 1D-Creep in single loading step

\[ C_\alpha = \text{secondary compression index} \]

\[ e_{eoc} = \text{void ratio at end of consolidation} \]

\[ t_{eoc} = \text{time at end of consolidation} \]
Early literature on secondary compression

Buisman (1936) used \( \varepsilon \) instead of \( e \):

\[
\varepsilon = \varepsilon_{eoc} + C_B \log \frac{t}{t_{eoc}} = \varepsilon_{eoc} + C_B \log \frac{t_{eoc} + t'}{t_{eoc}}
\]

- \( \varepsilon_{eoc} \) = end of consolidation strain
- \( t_{eoc} \) = end of consolidation time
- \( t' = t - t_{eoc} \) = creep time

Bjerrum (1967) and Garlanger (1972):

\[
e = e_{eoc} - C_a \log \frac{\tau + t'}{\tau} \quad \text{with} \quad C_a = (1 + e_0) \cdot C_B = \text{creep index}
\]

- \( \tau \) = extra parameter
Increase of OCR due to creep after Bjerrum (1967)

State of overconsolidation can be reached both by creep and unloading

\[ OCR = \frac{\sigma_p}{\sigma'_o} \]

\[ C_S = \text{swelling index} \]
OCR-contours are lines for constant rate of creep (isotaches)

\[ \dot{e} \equiv \frac{de}{dt} \]

NCL

OCR = 1 \quad \dot{e} = -a

OCR = 1.3 \quad \dot{e} = \text{small, e.g. } -a \cdot 10^{-3}

OCR = 1.7 \quad \dot{e} = \text{very small, e.g. } -a \cdot 10^{-6}

Den Haan (1994)
Oedometer creep model versus classical creep concepts

Oedometer creep model: \[ \dot{\varepsilon}^c = \text{constant} \cdot \left( \frac{1}{\text{OCR}} \right)^\beta \] for all values of \( \sigma \)

Classical concepts:

Norton (1929): \[ \dot{\varepsilon}^c = \frac{1}{\tau} \left( \sigma - \sigma_o \right)^\beta \] \( \tau = \) temperature dependent

Prandtl (1928): \[ \dot{\varepsilon}^c = \frac{1}{\tau} \cdot \sin (\beta \sigma - \beta \sigma_o) \] for \( \sigma > \sigma_o \)

Soderberg (1936): \[ \dot{\varepsilon}^c = \frac{1}{\tau} \cdot \left( \exp (\beta \sigma - \beta \sigma_o) - 1 \right) \] for \( \sigma > \sigma_o \)

Also read:
Oedometer creep model

\[ \dot{e} = \dot{e}^e + \dot{e}^c = \frac{-C_s}{\ln 10} \frac{\dot{\sigma}'}{\sigma'} + \frac{-C_\alpha}{\ln 10} \frac{1}{\tau} \left( \frac{\sigma'}{\sigma_p} \right) \]

Typical soil data: \( C_s \approx \frac{C_c}{10} \) and \( C_\alpha \approx \frac{C_c}{30} \) \( \Rightarrow \frac{C_c - C_s}{C_\alpha} \approx 27 \)

It follows that the creep rate is proportional to

\[ \left( \frac{\sigma'}{\sigma_p} \right)^{27} = \frac{1}{\text{OCR}^{27}} \]

It follows that the creep rate is negligibly small for OCR well beyond unity.

\( C_c = \) compression index
\( \tau = \) usually 1 day, but temperature dependent
Tremendous influence of overconsolidation ratio on creep rate

Example: \( C_c = 0.15 \quad C_s = 0.015 \quad C_\alpha = 0.005 \)

\[
\dot{e}^c = \frac{-C_\alpha}{\tau \ln 10} \left( \frac{\sigma'}{\sigma_p} \right)^{\frac{C_c-C_s}{C_\alpha}} = \frac{-a}{OCR^{27}}
\]

<table>
<thead>
<tr>
<th>OCR</th>
<th>( \dot{e}^c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>(- a )</td>
</tr>
<tr>
<td>1.3</td>
<td>(- a \cdot 10^{-3} )</td>
</tr>
<tr>
<td>1.7</td>
<td>(- a \cdot 10^{-6} )</td>
</tr>
</tbody>
</table>
Creep model for isotropic loading \((\sigma_1 = \sigma_2 = \sigma_3)\)

\[
\dot{e} = \dot{e}^e + \dot{e}^c = -\kappa \frac{\dot{p}'}{p'} - \mu \frac{1}{\tau} \left( \frac{p'}{p_p} \right)^{\frac{\lambda - \kappa}{\mu}}
\]

\[
p' = \frac{1}{3} (\sigma_1' + \sigma_2' + \sigma_3')
\]

\[
\lambda = C_c / \ln 10 = \text{modified compression index}
\]

\[
\kappa \approx 2C_s / \ln 10 = \text{modified swelling index}
\]

\[
\mu = C_\alpha / \ln 10 = \text{modified creep index}
\]

The preconsolidation pressure \(p_p\) must be continuously updated by using

\[
\dot{p}_p = -\frac{p_p}{\lambda - \kappa} \dot{e}^c
\]

\(p_p = \text{isotropic preconsolidation pressure}\)

\[
p_p = \frac{p_{pe}}{\lambda - \kappa} \cdot \exp (e_0^c - e^c)
\]
Part 2

3D isotropic creep model
or
Soft Soil Creep Model

Publications on this model:


Ellipses of Modified Cam Clay are contours for constant rate of volumetric strain.

\[ p'_{eq} = p'_{equivalent} = p' + \frac{q^2}{M^2p'} \]

NCL: \[ p'_{eq} = p_p \]

\( M = \) slope of critical state line

MCC model has yield function \( f = p'_{eq} - p_p \)

\( q = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \)
3D creep model with isotropy

\[ e = \dot{e}^e + \dot{e}^c = -\kappa \frac{\dot{p}^'}{p} - \frac{\mu}{\tau} \left( \frac{p_{eq}^'}{p_p} \right)^\frac{\lambda - \kappa}{\mu} \]

with \[ \dot{p}_p = \frac{-p_p}{\lambda - \kappa} \dot{e}^c \]

For 3D we use the equivalent mean stress:

\[ p_{eq}^' = p^' + \frac{q^2}{M^2 p^'} \]
Modified Cam Clay model versus Soft Soil Creep model

Both models have:
\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c \]

Both models have hardening:
\[ \dot{p}_p = -\frac{p_p}{\lambda - \kappa} \dot{\varepsilon}^c \]

Both models have flow rule:
\[ \dot{\varepsilon}_i^c = \Lambda \frac{\partial p'_{eq}}{\partial \sigma_i} \]

MCC: increase of density by primary loading:
\[ \dot{\varepsilon}^c = -\frac{\lambda - \kappa}{p_p} \frac{\partial p'_{eq}}{\partial \sigma_i} \dot{\sigma}_i \] (summation convention)

NCL is yield locus

SSC: increase of density by creep:
\[ \dot{\varepsilon}^c = -\frac{\mu}{\tau} \left( \frac{p'_{eq}}{p_p} \right)^{\frac{\lambda - \kappa}{\mu}} \]

NCL is creep rate locus

\[ \dot{\varepsilon}_{volumetric} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = \Lambda \cdot \left( \frac{\partial p'_{eq}}{\partial \sigma_1} + \frac{\partial p'_{eq}}{\partial \sigma_2} + \frac{\partial p'_{eq}}{\partial \sigma_3} \right) = \Lambda \cdot \left( 1 - \frac{q^2}{M^2 p'^2} \right) = \Lambda \cdot d \]
The equations of the Soft Soil Creep model

\[ \dot{\epsilon}_i = \dot{\epsilon}_i^e + \dot{\epsilon}_i^c = C_{ij}^e \cdot \dot{\sigma}_j + \Lambda \cdot \frac{\partial p'_{eq}}{\partial \sigma_i} \]

\[ \Lambda = \frac{1}{d} \frac{\mu^*}{\tau} \left( \frac{p'_{eq}}{p_p} \right)^{\frac{\lambda - \kappa}{\mu}} \]

\[ d = 1 - \left( \frac{q/p'}{M} \right)^2 \]

elasticity matrix \[ C_{ij}^e = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \]

\[ E = 3(1 - 2\nu) \frac{p'}{\kappa^*} \]

\[ p'_{eq} = p' + \frac{q^2}{M^2 p'} \]

\[ p_p = p_p^0 \exp \left( \frac{e_0^c - e^c}{\lambda - \kappa} \right) \]

\[ \mu^* = \mu/(1 + e_0) \quad \kappa^* = \kappa/(1 + e_0) \quad \nu = \text{Poisson ratio} \]

Model parameters: \( \lambda, \kappa, \mu, \nu, M \) \( (\tau = 1 \text{ day for temperature of } 10^\circ\text{C}) \)

Initial conditions: \( \sigma_{1o}', \sigma_{2o}', \sigma_{3o}', p_{p0}, e_0 \)
Performance of SSC model for drained triaxial tests on NC-soil

\[ M = \frac{6 \sin \varphi_{cs}}{3 - \sin \varphi_{cs}} \]
Performance of SSC model for undrained triaxial tests on NC-soil
Influence of strain rate on the undrained shear strength in triaxial compression

Part 3

3D anisotropic creep model

Important publications:


Tests on anisotropically consolidated clays

Contour lines for constant rate of volumetric creep strain rate after Boudali (1995)

Equivalent mean stress for anistropic creep model

\[ \dot{\varepsilon}^c = - \frac{\mu}{\tau} \left( \frac{p_{\text{eq}}'}{p_p} \right)^{\frac{\lambda - \kappa}{\mu}} \]

\[ p_{\text{eq}}' = p' + \frac{(q - \alpha p')^2}{M^2 - \alpha^2} \frac{1}{p'} \]

\( \alpha = 0 \) gives isotropic model

\( \alpha_o \) relates to \( K_o \)

Wheeler et al. (1993): \( \dot{\alpha} = m \left[ \left( \frac{3q}{4p'} - \alpha \right) \langle \dot{\varepsilon}_v^p \rangle + \beta \left( \frac{q}{3p'} - \alpha \right) |\dot{\varepsilon}_d^p| \right] \)
The equations of the anisotropic creep model

\[ \dot{\varepsilon}_i = \dot{\varepsilon}_i^e + \dot{\varepsilon}_i^c = C_{ij}^e \cdot \dot{\sigma}_j + \Lambda \cdot \frac{\partial p'_{eq}}{\partial \sigma_i} \]

\[ \Lambda = \frac{1}{d} \frac{\mu^*}{\tau} \left( \frac{p'_{eq}}{p_p} \right) \frac{\lambda - \kappa}{\mu} \]

\[ d = 1 - \frac{1}{p'^2} \frac{q^2 - \alpha^2 p'^2}{M^2 - \alpha^2} \]

**elastcity matrix**

\[ C_{ij}^e = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \]

\[ E = 3 (1 - 2 \nu) \frac{p'}{\kappa^*} \]

\[ p'_{eq} = p' + \frac{(q - \alpha p')^2}{M^2 - \alpha^2} \frac{1}{p'} \]

\[ \dot{p}_p = \frac{-p_p}{\lambda - \kappa} \dot{\varepsilon}_c \]

**Model parameters:** \( \lambda, \kappa, \mu, \nu, M \)

**Initial conditions:** \( \sigma_{i0}', \sigma_{20}', \sigma_{30}', \alpha', p_{po}, e_o \)
Practical difference between isotropic and anisotropic model

Isotropic creep model predicts a relatively low creep rate in oedometer tests.

Anisotropic creep model predicts a relatively high creep rate in oedometer tests.

On calibrating the isotropic model by oedometer data one overestimates the creep in all other stress paths. This is exactly our experience with the isotropic creep model. On using the anisotropic creep model we expect realistic predictions.
Concluding remarks

1. The isotropic creep model is a straightforward generalization of MCC to include creep
2. Creep tends to be overpredicted by the isotropic creep model
3. Anisotropic creep model would seem to be more realistic
4. Laboratory creep tends to be larger than field creep. Therefore lab data may need corrections (Leroueil, 1997)

We are working on numerical implementation, validation and verification of the anisotropic model.

Undrained triaxial tests after Vaid et al. (1977) and simulation with SSC model