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CABLE STRUCTURE WITH LOAD-ADAPTING GEOMETRY

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ABSTRACT: For an application in a composite cable-stayed footbridge a system is developed, which is to ensure an equal distribution of static or quasi-static life loads in the composites retainers. Thus the allowed life load can be maximized for this kind of structures while maintaining the necessary wide safety margin. The structure's optimal geometry for the control procedure is determined by means of an algorithmic formfinding process, based on the method of force density. The results of the shape optimization seem to match with a mechanical device to be developed.

1. INTRODUCTION

The fragile behaviour of composite materials (carbon and glass fibres) and the lack of experience require high safety factors, which are prejudicial to the introduction of composites in civil engineering. The study's context is an all-composite cable-stayed footbridge for which a shape control system is developed. The shape of the footbridge will adapt itself to external life-loads to equalize as much as possible the stresses within the different types of elements. As a result the allowable life loads can be maximized even if high safety factors are to be kept.
This paper presents briefly the method of force density. Then this method is applied in an algorithmic form to a simplified 2D structure inspired by the final cable-stayed footbridge. Finally the optimisation results are interpreted for an application to a mechanical control device.

2. FORCE DENSITY METHOD

With the force density method, introduced by Linkwitz in 1971 [1,2], equilibrium shapes of prestressed cable-structures can easily be found. As a highly non-linear system of equations is linearised, this method is proved to be highly efficient when determining consistent equilibrium forms for tension nets.
The method of force densities is briefly introduced means a simplified structure whose geometry is chosen to be close to the footbridge's geometry we plan to study. Fig. 1 shows the Finite Element model created with ANSYS© for the cable-stayed footbridge. Fig. 2 shows the simplified model chosen for the shape optimisation with Scilab and the notations used.

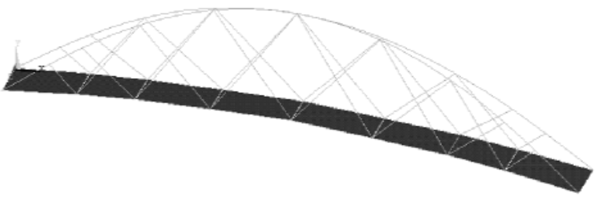

Figure 1 – FE-Model in 3D.

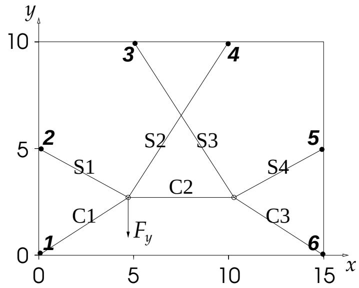

Figure 2 – Simplified model with notations

The simplified structure consists of four stays (S1, S2, S3 and S4) and a cable made of three parts (C1, C2 and C3). Both element types work in tension only. Points 1, 2, 3, 4, 5 and 6 are constrained in both x- and y-direction, thus their coordinates are known. We will determine then the coordinates of points 7 and 8 according to the equilibrium shape for a load set. The structure loads we consider are prestress and eventually a force on nodes 7 and/or 8. In the following equations we chose to apply a vertical load F_y on node 7.

2.1 The equations of the method of force density

Considering the structures equilibrium on nodes 7 and 8, we can write down the following equations:

$$\text{Node 7}\quad \begin{cases} (x_7 - x_1)\dfrac{f_{(17)}}{l_{(17)}} + (x_7 - x_2)\dfrac{f_{(27)}}{l_{(27)}} + (x_7 - x_4)\dfrac{f_{(47)}}{l_{(47)}} + (x_7 - x_8)\dfrac{f_{(78)}}{l_{(78)}} = 0 \\ (y_7 - y_1)\dfrac{f_{(17)}}{l_{(17)}} + (y_7 - y_2)\dfrac{f_{(27)}}{l_{(27)}} + (y_7 - y_4)\dfrac{f_{(47)}}{l_{(47)}} + (y_7 - y_8)\dfrac{f_{(78)}}{l_{(78)}} = F_y \end{cases} \qquad (1)$$

$$\text{Node 8}\quad \begin{cases} (x_8 - x_7)\dfrac{f_{('78)}}{l_{(78)}} + (x_8 - x_3)\dfrac{f_{(38)}}{l_{(38)}} + (x_8 - x_5)\dfrac{f_{(58)}}{l_{(58)}} + (x_8 - x_6)\dfrac{f_{(68)}}{l_{(68)}} = 0 \\ (y_8 - y_7)\dfrac{f_{(78)}}{l_{(78)}} + (y_8 - y_3)\dfrac{f_{(38)}}{l_{(38)}} + (y_8 - y_5)\dfrac{f_{(58)}}{l_{(58)}} + (y_8 - y_6)\dfrac{f_{(68)}}{l_{(68)}} = 0 \end{cases} \qquad (2)$$

with:
- x_k x -coordinate of node k
- y_k y-coordinate of node k
- $f_{(jk)}$ force within the element between node j and k
- $l_{(jk)}$ length of the stress-extended element between node j and k

The distance between node j and k, i.e. the length of the stress-extended element, is described using Pythagoras' law $l_{(jk)} = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}$. The system of equations given by (1) and (2) is then obviously highly non-linear regarding the geometrical variables (x_7, y_7, x_8 and y_8). Introducing force densities defined as $q_{(jk)} = f_{(jk)} / l_{(jk)}$ the equations (1) and (2) can be linearized as shown in equations (3) and (4).

$$\text{Node 7}\quad \begin{cases} x_7(q_{(17)} + q_{(27)} + q_{(47)} + q_{(78)}) - x_8 q_{(78)} = x_1 q_{(17)} + x_2 q_{(27)} + x_4 q_{(47)} \\ y_7(q_{(17)} + q_{(27)} + q_{(47)} + q_{(78)}) - y_8 q_{(78)} = y_1 q_{(17)} + y_2 q_{(27)} + y_4 q_{(47)} + F_y \end{cases} \qquad (3)$$

$$\text{Node 8}\quad \begin{cases} -x_7 q_{(78)} + x_8(q_{(78)} + q_{(38)} + q_{(58)} + q_{(68)}) = x_3 q_{(38)} + x_5 q_{(58)} + x_6 q_{(68)} \\ -y_7 q_{(78)} + y_8(q_{(78)} + q_{(38)} + q_{(58)} + q_{(68)}) = y_3 q_{(38)} + y_5 q_{(58)} + y_6 q_{(68)} \end{cases} \qquad (4)$$

with: $q_{(jk)}$ the force density of the element between nodes j and k, i.e. $f_{(jk)}/l_{(jk)}$.

We obtain a linear system of equations with 4 equations and 4 unknowns (x_7, y_7, x_8 and y_8). A particular set of force densities $q_{(jk)}$ relates to a unique equilibrium shape.

2.2 Application to the prestressed structure

As Linkwitz noticed [3], investigations and practical experiments justify the choice of very simple 'types' of force densities to create equilibrium shapes as an initial approach to formfinding: a force density q = c is assigned to elements of equal length while a force density of q = c / l is used for structures with elements of irregular length. In both cases c is a constant value, but while the constant c means a constant force density when using the first 'type' of force densities, the constant c stands for a constant force when applying force densities of the second type. The structure we study has elements of irregular length. Hence for the shape optimization the force densities q = c / l of the second type are chosen.
Two different sets of force densities q_s for the stays and q_c for the cable elements are chosen, i.e. a constant force $c_s = f_s$ for all the stays and a constant force $c_c = f_c$ for the cable. These force densities state a prestress distribution within the structure. Remembering the force density definition q = f / l, it becomes obvious that implementing a non-zero force density to an element results in an equilibrium form with a non-zero force for this element. Thus all elements are subjected to a force. The two different sets of force densities q_s / q_c form two different states of prestress distribution. No force F_y is applied.

The equilibrium forms for the initial shape 1 are given in Fig 3 and Fig 4. The force densities chosen for Fig. 3 are q_s = 1 / l and q_c = 2 / l, with l relating to the length of the stress-extended element in the initial geometry. The equilibrium shape of Fig. 4 results from choice of the force densities q_s = 1 / l and q_c = 10 / l. In Fig. 5 the ratio of q_s / q_c = 1 / 10 is preserved but the equilibrium is calculated for an initial shape 2. The initial geometry is drawn in a dotted line and the equilibrium form is represented as an unbroken line.

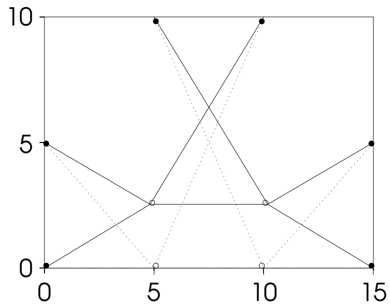

Figure 3 – Initial shape 1 and equilibrium for
$q_s = 1/l$ **and** $q_c = 2/l$.

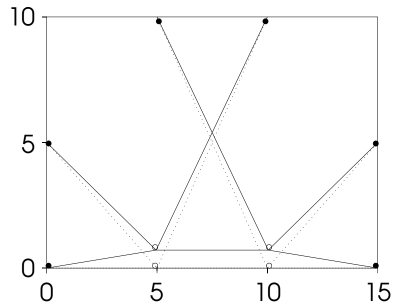

Figure 4 – Initial shape 1 and equilibrium for
$q_s = 1/l$ **and** $q_c = 10/l$.

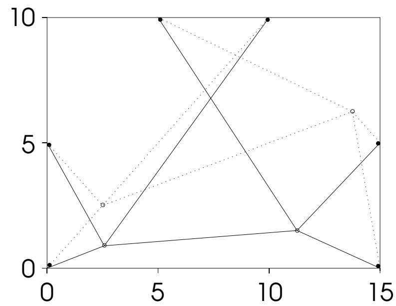

Figure 5 – Initial shape 2 and equilibrium for
$q_s = 1/l$ **and** $q_c = 10/l$.

It can be seen that:

- the force density ratio controls the 'radius of curvature' of the cable (elements C1, C2 and C3).
- the initial shape has an influence on the equilibrium shape. The information of the initial shape is not included through the coordinates of the 4 unknown (x_7, y_7, x_8 and y_8) in equations (3) and (4) but through the lengths of each element in the force densities q. Actually, to determine the equilibrium shape, the force densities are calculated depending on the initial extended length of the elements and evidently different initial geometries match with different initial element lengths.

3. ITERATIVE STRESS-EQUALIZING METHOD

The chosen algorithm is very simple. Other types have been proposed by [4,5] but as the aim is not to develop a highly efficient and fast algorithm but a stress control device, this approach seems justifiable.
The aim of the presented algorithm is to determine geometries providing a (quasi-)equal stress distribution within each type of element. As each type of element has a determined cross-section, to equalize stresses means to equalize forces within the elements no matter what their length.
Fig. 6 shows the iteration process chosen; troughout all the iterations each c (c_s and c_c) is kept constant and thus corresponds to the equalized forces f_s and f_c to be reached (if possible) within the elements.

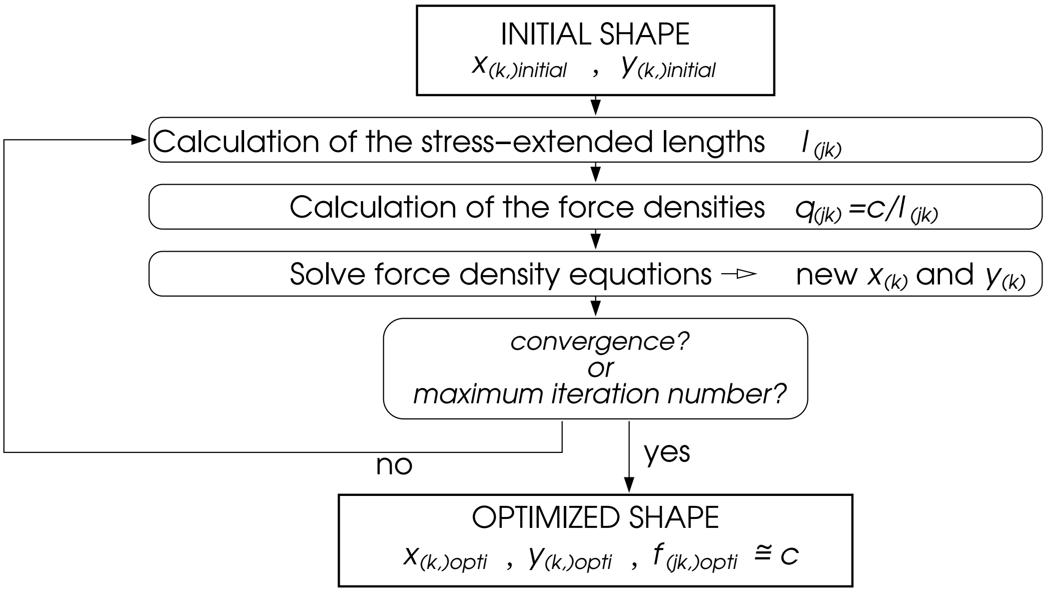

Figure 6 – Iteration process to generate equilibrium forms equalizing stresses within elements.

However, it should be noticed that it is not always feasible to determine a shape that leads to the imposed forces in each element. Nevertheless the iterative process can determine a shape as close as possible to the requirements; this will be the optimized geometry. This is why two classical criteria are tested to stop the iteration procedure. On the one hand we have a convergence test and on the other hand a maximum iteration step.

3.1 Application 1: prestressed structure

For this evaluation the initial shape 2 is chosen because of the greater variation of the element's length. This allows an easier evaluation of the influence of the initial geometry. Fig. 7 shows the improvement of the equilibrium shape for $q_s = 1 / l$ and $q_c = 10 / l$ throughout the iterations, i.e. iterations i = 1, 2, 10. Table 2 gives the maximum element length difference of all elements for these iterations. This maximum difference dl_i^{max} is the maximum difference between the distance of nodes j_i and k_i (or the stress extended length of the element between nodes j_i and k_i) of iteration i and the distance of nodes j_{i-1} and k_{i-1} of the previous iteration step i - 1, i.e. $dl_i^{max} = \max\left|l_{(jk,)i} - l_{(jk,)i-1}\right|$. Studying the maximum length difference, the improvement of the equilibrium geometry can be observed. When the new geometry does not particularly differ from the previous one, i.e. the maximum length difference dl_i^{max} gets close to zero, the force densities, depending on the lengths, do not vary much either. The algorithm converges. This can easily be noticed in Table 2 on the fact that the average forces of the stays $\bar{f}_{s,i} \cong c_s = 1$ and of the cable elements $\bar{f}_{c,i} \cong c_c = 10$ quite reach the required forces c in iteration step i = 10. Additionally the standard deviations $s_{s,i}$ and $s_{c,i}$. indicate that a good equalization of the forces is archived for i = 10.

Table 2 – Trends of dl_i^{max}, $\bar{f}_{s,i}$ and $\bar{f}_{c,i}$, $s_{s,i}$ and $s_{c,i}$

i	dl_i^{max}	$\bar{f}_{s,i}$	$s_{s,i}$	$\bar{f}_{c,i}$	$s_{c,i}$
1	1.896	1.621	0.859	7.127	0.723
2	0.136	1.074	0.048	9.803	0.157
10	0.013	1.000	0.002	10.024	0.093

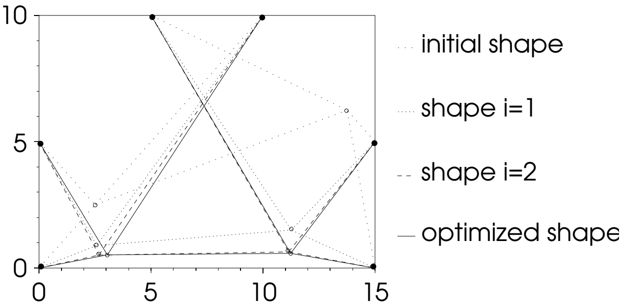

Figure 7 – Improved equilibrium shape for iteration steps i = 1, 2, 10.

Considering the evolution of the equilibrium geometry throughout iteration steps, it can be fixed that:

- the greatest variation of shape occurs between initial geometry and equilibrium geometry of the first iteration step i = 1.
- the optimized geometry is determined by the initial shape. This can be noticed considering that even though a 'symmetrical' prestress load is applied the lengths of elements C1 and C3 are not equal.

3.2 Application 2: loaded and prestressed structure

The influence of external load on the optimized shape of the prestressed structure is now studied. As noticed in the previous chapter, the initial geometry is significant when determining new optimized shapes. This is why the initial prestress shape of the structure has to be chosen carefully. The selection procedure for an optimized initial shape will not further be discussed in this paper, but it should be kept in mind that this detail will lead to additional investigation.
In this paper, the selected optimized prestressed shape is the equilibrium form shown in Fig. 2. The force density ratio that leads to this optimized shape is close to the ratio resulting from the previous Finite Element study of the footbridge. The structure is loaded in two different manners.
First load case (LC1): The structure is loaded symmetrically with a load $F_y = 10$ applied on nodes 7 and 8. These forces model the pedestrian load. To take into account the new internal stress distribution resulting from external loading the adapted force densities allotted are $q_s = 10 / l$ and $q_c = 25 / l$. These force densities are also determined based on the Finite Element computation results.
Second load case (LC2): The structure is loaded asymmetrically with $F_y = 10$ applied only on node 7. The matching force densities are $q_s = 5 / l$ and $q_c = 20 / l$.
The Fig. 8 and 9 illustrate the equilibrium shapes for both load cases at iteration step i = 10. As for the previous figures, the dotted line shows the initial geometry and the unbroken line the equilibrium form.

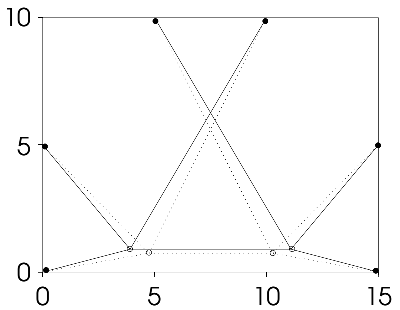

Figure 8 – Equilibrium for LC1 with $q_s = 10/l$ and $q_c = 25/l$.

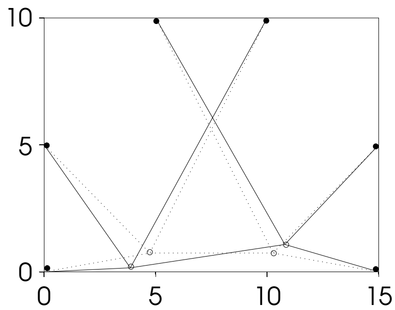

Figure 9 – Equilibrium for LC2 with $q_s = 5/l$ and $q_c = 20/l$

The initial stress-extended length $l_{prestress}$ and the stress-extended additional element length Δl_{LC} each element requires to create the equilibrium geometry are displayed in Table 3 for both load cases. The additional stress-extended element length is $\Delta l_{LC} = l_{LC} - l_{prestress}$. The additional element length $\Delta l_{LC(,C)}$ of the entire cable is calculated by $\Delta l_{LC(,C)} = \Delta l_{LC(,C1)} + \Delta l_{LC(,C2)} + \Delta l_{LC(,C3)}$.

Table 3 – Initial stress-extended length and required additional stress-extended element length to create optimized shape.

Element	S1	S2	S3	S4	C1	C2	C3	Cable
$l_{prestress}$	6.371	10.650	10.650	6.371	4.795	5.525	4.795	15.115
Δl_{LC1}	-0.708	0.304	0.304	-0.708	-0.789	1.667	-0.789	0.089
Δl_{LC2}	-0.176	0.946	-0.013	-0.620	-0.927	1.464	-0.453	0.085

4. EVALUATION OF A CONTROL DEVICE MOCK-UP

All the lengths given in the previous chapters are lengths of stress-extended elements. The optimization approach takes into account neither the geometry of the elements nor the element's material properties. In fact with the force density method only the structure's geometry is adapted to create a force flow matching with a determined force distribution.
The next step is the construction of a mock-up of the simplified structure to validate the numerical results and to start developing a control device. When determining the 'cutting pattern' of the mock-up, the slack lengths of the elements are needed. It becomes necessary to consider the element's geometry and material. In order to make the notations clear, the Fig. 10 shows graphically all notations used for the lengths.

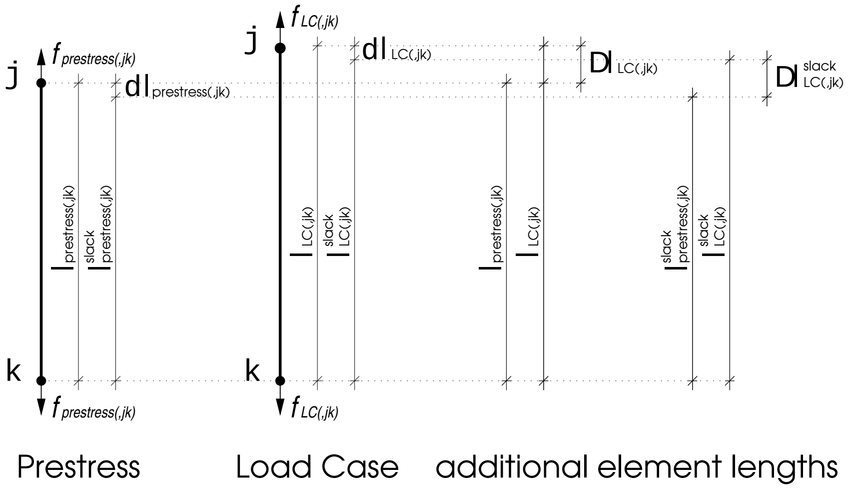

Figure 10 – Notations of lengths $l_{(jk)}$ and $l^{slack}_{(jk)}$, elongations $\delta l_{(jk)}$ and additional lengths $\Delta l_{(jk)}$ and $\Delta l^{slack}_{(jk)}$ .for prestress and load cases.

Considering Hooke's linear law to be applicable for the element's material of this structure, the slack lengths $l^{slack}_{(jk)}$ of the elements can easily be determined by the following equations:

$$l_{(jk)} = l^{slack}_{(jk)} + \delta l_{(jk)} \quad (5)$$

$$\delta l_{(jk)} = \frac{l^{slack}_{(jk)}}{E_E A_E} f_{(jk)} = \frac{l^{slack}_{(jk)}}{E_E A_E} c_E \quad (6)$$

$$l_{(jk)}^{slack} = l_{(jk)} \cdot \left(1 + \frac{c_E}{E_E A_E}\right)^{-1} \quad (7)$$

with:

$l_{(jk)}$	length of the stress-extended element between node j and k
$l_{(jk)}^{slack}$	slack length of the element between node j and k
$\delta l_{(jk)}$	elastic elongation of the element between node j and k
$f_{(jk)}$	force within the element between node j and k
E_E	stiffness of the material of the element type E
A_E	area of the cross section of the element type E
c_E	constant matching with the imposed force for element E

Table 4 shows the slack length for each element in the initial state of prestress called $l_{prestress}^{slack}$, and for both load cases called l_{LC}^{slack}. The additional slack element length Δl_{LC}^{slack} of each element for each load case (LC) is also calculated with $\Delta l_{LC}^{slack} = l_{LC}^{slack} - l_{prestress}^{slack}$. According to the FE-model of the footbridge the stiffness of the stays and of the cable is $E_S = E_C = 40\ 000$ MPa, the cross-section of the stays is $A_S = 0.0001\ m^2$ and the cross-section of the cable is $A_C\ m^2$. To match with the dimensions, all lengths in Table 4 are given in meters.

Table 4 – Slack lengths l^{slack} and required slack additional length Δl_{LC}^{slack} to create optimized shape.

Element	S1	S2	S3	S4	C1	C2	C3	Cable
$l_{prestress}^{slack}$	6.369	10.647	10.647	6.369	4.795	5.525	4.795	15.114
l_{LC1}^{slack}	5.649	10.927	5.649	10.927	4.006	7.192	4.006	15.204
Δl_{LC1}^{slack}	-0.720	0.280	-0.720	0.280	-0.789	1.667	-0.789	0.090
l_{LC2}^{slack}	6.187	11.281	10.624	5.744	3.868	6.989	4.342	15.199
Δl_{LC2}^{slack}	-0.182	0.634	-0.023	-0.625	-0.927	1.464	-0.453	0.085

Because of the symmetry of LC1, the slack length and additional length of the element S3 are identical to those of element S2, and similarly element S4 corresponds to element S1, and C3 to C1.
The greatest stricture needed is the one to form the equilibrium shape of LC1. The element S1 (and the element S4) has to be shortend by $-\Delta l_{LC1(,S1)}^{slack} = 0.720$m slack length.
The most important additional length is required for the equilibrium geometry of LC2. Here the element S2 needs $\Delta l_{LC2(,S2)}^{slack} = 0.634$m extra slack length.
The cable remains almost the same length: the maximum additional length is $\Delta l_{LC1(,C)}^{slack} = 0.090$m.

Considering the application on the 3D-footbridge, the necessary additional slack length will probably be smaller because of a greater number of retainers and cable elements. The results of this first evaluation allow further reflections; it should be possible to design a control device able to adapt the element's length within these margins.

5. CONCLUSIONS

The final aim is to control the stresses within elements of a loaded structure by adapting the shape. In the first part of this paper a short synopsis of the force density method was given. Then a basic iteration process was presented. The forces in the different element types are to be equalized adjusting the lengths of the elements to the load situation. In the third part a preliminairy approach allowed an evaluation of the element elongations required for this adaptive shape structure. The order of magnitude of the calculated elongations seemed suitable to the whole geometry and applying such displacements on the structure should be possible with a mechanical device to be developed.

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